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Comparative study of the reliability of some deterministic and stochastic interpolation methods for ground bearing capacity mapping: Case of the Olembé social housing site in Cameroon

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Abstract

This paper deals with the comparative study of the reliability of some deterministic and stochastic interpolation methods for ground bearing mapping. Covering an area of 121,481.65 m², the site covered by this study is located in the locality of Olembé in the central region of Cameroon. The methodology consisted of collecting from the Ministry of Housing and Urban Development the results of the tests relating to the bearing capacity of the soil for depths of 1.2m, 2.4m and 4.5m as well as the location of the sounding points. Using ARCGIS mapping software, ground bearing mapping was performed by Inverse Distance Weighting (IDW) and Local Polynomial Interpolation (LPI) as deterministic interpolation methods while Ordinary Kriging (KO) and Empirical Bayesian Kriging (KBE) were retained as stochastic methods. The comparative analysis of the interpolation error according to the different methods studied shows that the stochastic methods are the most precise and the empirical Bayesian kriging is the most precise of all. The interpolation error between 8.1% and 41.20% for the deterministic methods while for the stochastic methods it is between 0.4% and 8.80%. The minimum average of bearing capacity recorded for the deterministic methods is 2.35 bars, 3.52 bars, 1.89 Against 1.58bars, 2.42bars and 1.75 bars for the stochastic methods at a depth of 1.2m, 2.4m and 4.5m respectively.

Keywords: Cartography, Bearing capacity, Interpolation, Deterministic, Stochastic.

1. Introduction

An important step before the realization of the civil engineering works is the design of the foundation system of the work. According to the NF P 94-500 standard, the said design is used to develop geotechnical campaigns ranging from mission G1 to mission G4 for studies, mission G5 being concerned with the diagnosis of structures.

In the preliminary study phase (mission G1), The installation of the sampling points is carried out by meshing the next site according to the precision sought.

The tests being carried out only on specific points it is not possible to carry them out at any point of the site because this would require an infinity of sampling points. It is therefore necessary to develop tools to estimate the value of

the data studied between the real sampling points (Setianto and Triandini, 2020). Interpolation is one possible approach.

Interpolation is a consistent technique to estimate the value of a quantity at one site from samples of that quantity size harvested at other sites (Bossler, 2011).

Being usually divided into two groups, several interpolation methods exist and offer a different accuracy of the results depending on the object of study. It is therefore interesting to perform interpolations based on several methods in order to choose the one that is the most reliable in a specific study context.

Deterministic methods are based on purely mathematical properties, usually geometric, without taking into account the physical phenomenon that we are interested while stochastic methods use probabilistic models and derive from the statistical analysis of the data considered (Bossler, 2011).

Kriging is a stochastic spatial interpolation method that predicts the value of a natural phenomenon at unsampled sites, by a combination linear without bias and with minimum variance of the observations of the phenomenon in the neighboring sites (Baillargeon, 2005). There are several variants, including Ordinary Kriging (KO) and Empirical Bayesian Kriging (KBE).

Inverse distance weighted interpolation (IDW) and local polynomial interpolation (LPI) are part of the local deterministic interpolation methods. The first makes it possible to estimate the value at one point of the study area using the weighted average of the values of the points closest to the point considered (Leroux, 2007), while the second makes it possible to estimate said value by a locally polynomial function defined from the known values of the points (Leborgne, 2021).

Spatial interpolation is a tool used in different fields, including topography, meteorology, geodesy, and even geology.

This is the case of Fotios et al. (2013) who were interested in the kriging interpolation method for the estimation of the continuous spatial distribution of precipitation in Cyprus. Setianto and Triandini (2013) compared kriging interpolation methods and inverse distance weighting for lineament extraction. Rebai, Slama and Turk (2007) evaluated different interpolation methods (Inverse Distance, Minimal curvature, Natural Neighbor

Polynomial local Kriging) spatial for the production of a DTM in a GIS from topographic data collected in the Jebel Kechtilou-Jebel area Jebes in northern Tunisia.

Baillargeon et al. (2004) used the kriging method Statistical Interpolation multivariate data from precipitation in a hydrological modelling in the Gatineau River watershed in Quebec. Achilleos (2011) focused on the inverse distance-weighted interpolation method and error propagation mechanism for creating a DEM from an analogy topographical map.

The present study focuses on a comparative analysis between some stochastic and deterministic methods for ground bearing capacity mapping. These include Empirical Bayesian Kriging (KBE), Ordinary Kriging (KO), Inverse Distance weighted Interpolation (IDW) and Local Polynomial Interpolation (LPI).

2. Methodology

Extending over an area of 121,481.65 m², the study site is located in Cameroon, in the Centre region, MFoundi Division, Yaoundé1 subdivision at a place called Olembé. Figure (1) below illustrates the location of the site.

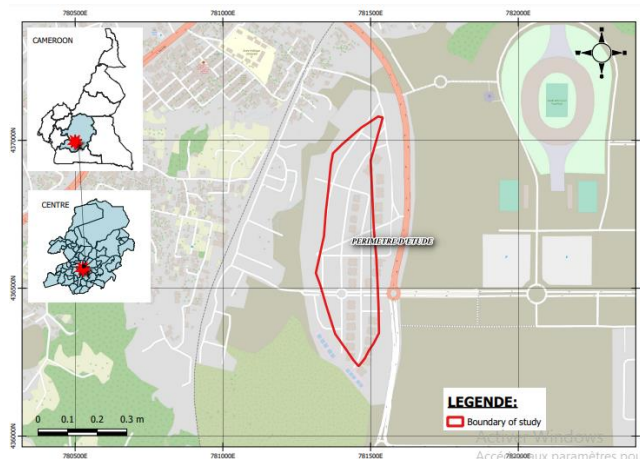


Figure 1 – Location of the site under study.

The methodology followed in this study consisted of collecting from the Ministry of Housing and Urban Development the results of the tests relating to the ground bearing capacity as well as the corresponding location at the sampling points.

At each sampling point, ground bearing data were retained for this study for depths of 1.2m, 2.4m and 4.5m. Using ArcGIS mapping software, ground bearing mapping was carried out using two

deterministic (IDW and LPI) and stochastic (KO and KBE) interpolation methods. Finally, a comparison of the reliability of the results obtained was made based on the interpolation error of each of them.

2.1. Soil bearing capacity

The soil bearing capacity was obtained thanks to the type B dynamic penetrometer test according to the NF P94-115 standard. Penetrometer probing consists of driving metal rods or tubes preceded by a spike into the ground, and this through blows given by a sheep of precise mass falling from a given height.

This test makes it possible to continuously measure the bearing capacity of the ground crossed, expressed by the resistance to dynamic penetration "Rpd" of a tip of diameter greater than that of the rod train used to drive the tip. The number of strokes (N) required to cross a given thickness of terrain is recorded, and the peak resistance to dynamic penetration is obtained through the formula:

$$R_p = \frac{M^2 \cdot H \cdot g}{A \cdot e(M + P)} \quad (1)$$

- M : mass of sheep;
- H : drop height;
- A : section of the tip;
- P : passive mass associated with stem weights;
- e : sinking by blow of sheep;
- g : acceleration of gravity.

Data on load-bearing capacity were collected from the Ministry of Housing and Urban Development of Cameroon.

2.2. Mapping of ground bearing capacity

The data processing was performed using ArcGIS version 10 software with spatial analysis tools using KBE, KO, IDW, and LPI methods.

2.3. Kriging

The kriging tool in ArcGIS software has an exploratory spatial data analysis module that

allows you to visualize and analyze data using statistical techniques. This module offers a wide range of possibilities for the detection of trends or drifts in the data, the identification of values and the study of the spatial correlations of these data (Koussa, 2018). The Figure (2) illustrates the steps involved in setting up kriging.

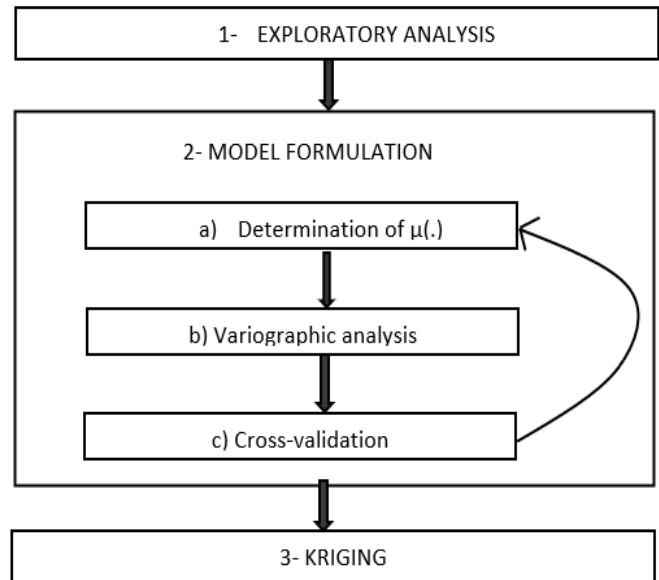


Figure 2 – Geostatistical methodology (Baillargeon, 2005).

The regionalized variable under study is a random function that breaks down as follows:

$$Z(s) = \mu(s) + \delta(s), s \in D \quad (2)$$

Or $\mu(\cdot)$ is the deterministic structure for expectation as a function of the location of observations and $\delta(\cdot)$ is a normal random function of zero expectation. The choice of the form of $\mu(\cdot)$ implies the adoption of a type of kriging:

For ordinary kriging: $\mu(s) = \mu$ is an unknown constant.

Empirical Bayesian kriging (KBE) automates the most difficult aspects of creating a valid kriging model. It differs from other kriging methods in that it takes into account the error introduced by the estimator of the underlying semivariogram (ESRI, 2022).

2.4. Inverse distance weighted (IDW)

In this method, the value to be estimated at a point in the study area is determined using the weighted average of the values of the points close to the point under consideration.

The generalized expression for finding an interpolated value $u(X)$ at a given point X using this method is an interpolation function:

$$u(X) = \frac{\sum_{k=0}^N w_k(X)^p u_k}{\sum_{k=0}^N w_k(X)^p}, \quad (3)$$

Where:

$$W_k(X) = \frac{1}{d(X, X_k)}, \quad (4)$$

$W_k(X)$ is a simple weighting function corresponding to the inverse of the distance between a known point X_k and the interpolation point X .

U_k is the value taken at a known point X_k .

The IDW can be adjusted to reduce interpolation error (Zhengquan et al., 2018)

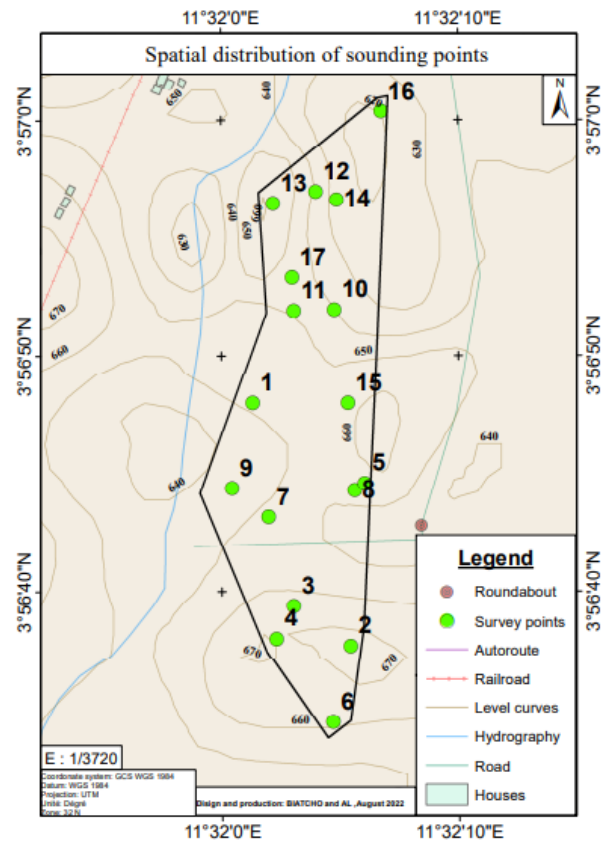


Figure 3 – Location of sampling points.

2.5. Local polynomial interpolation (LPI)

Considering $(n+1)$ points $(x_0; y_0), (x_1; y_1) \dots (x_n; y_n)$, of distinct two-to-two abscissa, polynomial interpolation consists in finding the unique polynomial $p(x) = a_n x^n + \dots + a_1 x + a_0$ degree at most n whose graph goes through the $(n+1)$ points (Boulier, 2020). This leads to the following system of equations:

$$\begin{pmatrix} x_0^n & \dots & x_0 & \dots & 1 \\ x_1^n & \dots & x_1 & \dots & 1 \\ \vdots & & \vdots & & \vdots \\ x_n^n & \dots & x_n & \dots & 1 \end{pmatrix} \cdot \begin{pmatrix} a_n \\ \vdots \\ a_1 \\ a_0 \end{pmatrix} = \begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_n \end{pmatrix} \quad (5)$$

3. Results and discussion

3.1. Survey points.

The following Figure (3) illustrates the location points of the sampling points that were performed.

Tables (1), (2) and (3) show the bearing capacity of the ground at the sampling points at 1.2m, 2.4m and 4.5m depth respectively.

Table 1 – Ground bearing capacity to a depth of 1.2m.

Pts	X	Y	Lift at 1,2m (bar)
P1	11,53361	3,946667	1,328146
P2	11,53475	3,943814	3,320364
P3	11,53409	3,944283	0,664073
P4	11,53388	3,943897	3,320364
P5	11,53492	3,945722	1,328146
P6	11,53454	3,942911	0,996109
P7	11,53379	3,945333	0,996109
P8	11,5348	3,945642	3,320364
P9	11,53337	3,945667	1,992218
P10	11,53457	3,947775	9,961092
P11	11,53409	3,947764	1,328146
P12	11,53435	3,949158	1,992218
P13	11,53385	3,949019	1,328146
P14	11,53459	3,949064	1,660182
P15	11,53472	3,946667	0,996109
P16	11,53512	3,9501	4,64851
P17	11,53408	3,948153	1,328146

Table 2 – Ground bearing capacity to a depth of 2.4m.

Pts	X	Y	Lift at 2,4m (bar)
P1	11,53361	3,946667	3,069618
P2	11,53475	3,943814	2,762656
P3	11,53409	3,944283	5,832274
P4	11,53388	3,943897	1,841771
P5	11,53492	3,945722	1,227847
P6	11,53454	3,942911	0,306962
P7	11,53379	3,945333	9,208854
P8	11,5348	3,945642	3,683541
P9	11,53337	3,945667	1,534809
P10	11,53457	3,947775	1,227847
P11	11,53409	3,947764	1,227847
P12	11,53435	3,949158	0,613924
P13	11,53385	3,949019	9,208854
P14	11,53459	3,949064	9,208854
P15	11,53472	3,946667	0,920885
P16	11,53512	3,9501	5,21835
P17	11,53408	3,948153	2,455694

Table 3 – Ground bearing capacity to a depth of 4.5m.

Pts	X	Y	Lift at 4,5m (bar)
P1	11,53361	3,946667	1,866782
P2	11,53475	3,943814	2,400149
P3	11,53409	3,944283	2,133466
P4	11,53388	3,943897	1,866782
P5	11,53492	3,945722	1,066733
P6	11,53454	3,942911	1,066733
P7	11,53379	3,945333	1,866782
P8	11,5348	3,945642	1,866782
P9	11,53337	3,945667	1,066733
P10	11,53457	3,947775	1,333416
P11	11,53409	3,947764	1,866782
P12	11,53435	3,949158	1,866782
P13	11,53385	3,949019	1,866782
P14	11,53459	3,949064	1,333416
P15	11,53472	3,946667	3,466882
P16	11,53512	3,9501	4,000248
P17	11,53408	3,948153	1,600099

3.2. Mapping of soil bearing capacity following the deterministic methods IDW and LPI.

3.2.1 Soil bearing capacity to a depth of 1.2m.

Figures (4) and (5) illustrate the mapping of ground bearing capacity at a depth of 1.2 m using IDW and LPI methods.

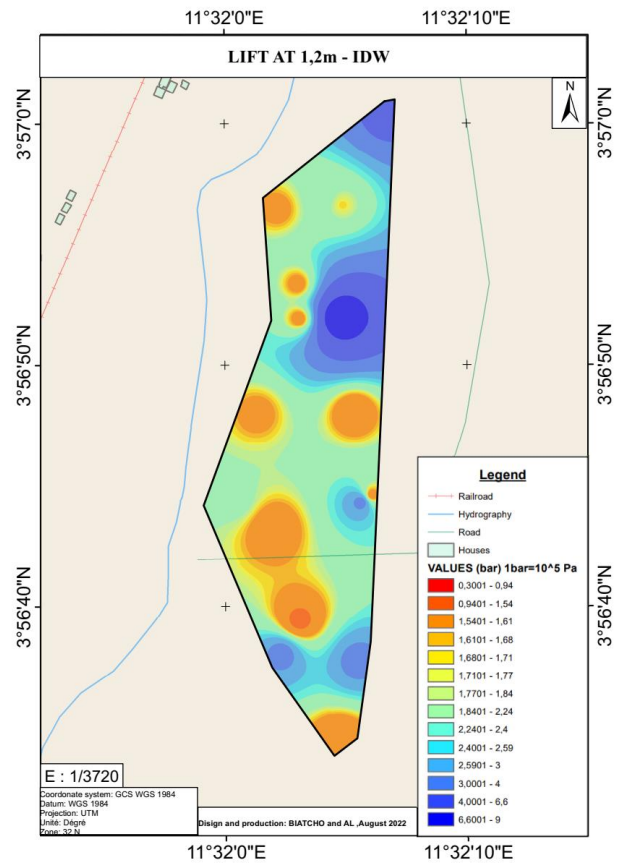


Figure 4 – Ground bearing capacity to a depth of 1.2m (IDW).

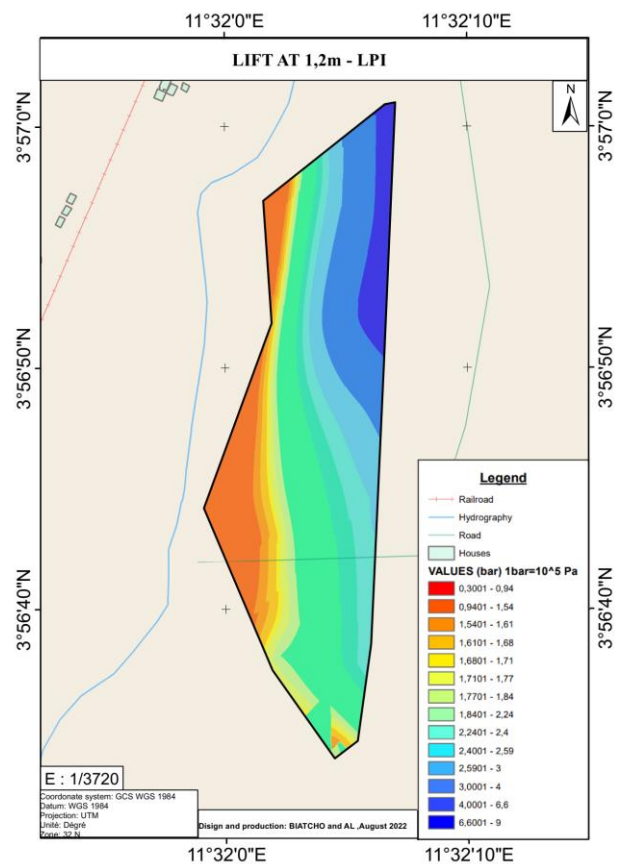


Figure 5 – Ground bearing capacity to a depth 1.2m (LPI).

3.2.2 Soil bearing capacity to a depth of 2.4m.

Figures (6) and (7) illustrate the mapping of ground bearing capacity at a depth of 2.4 m using IDW and LPI methods.

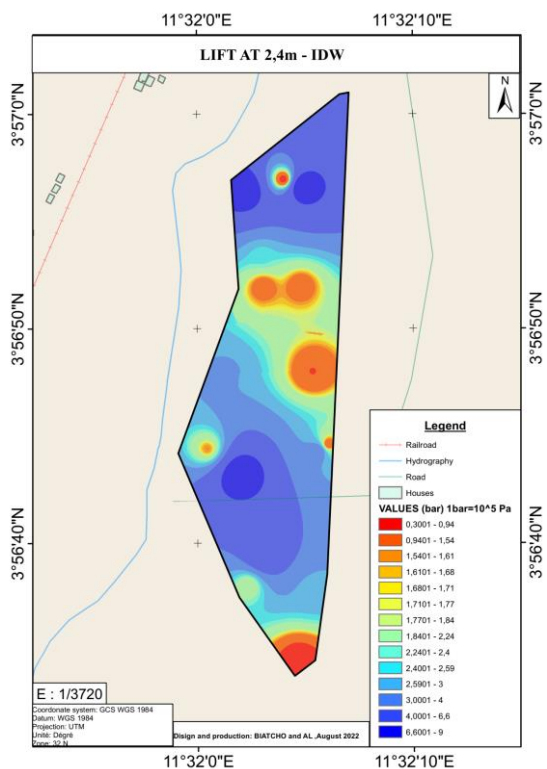


Figure 6 – Ground bearing capacity to a depth of 2.4m (IDW).

3.2.3 Soil bearing capacity to a depth of 4.5m.

Figures (8) and (9) illustrate the mapping of ground bearing capacity at a depth of 4.5 m using THE IDW and LPI methods.

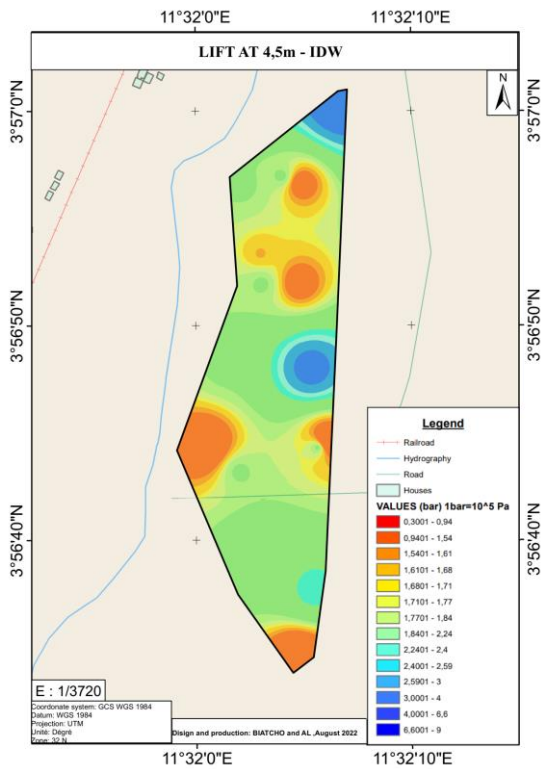


Figure 8 – Ground bearing capacity to a depth of 4.5m (IDW).

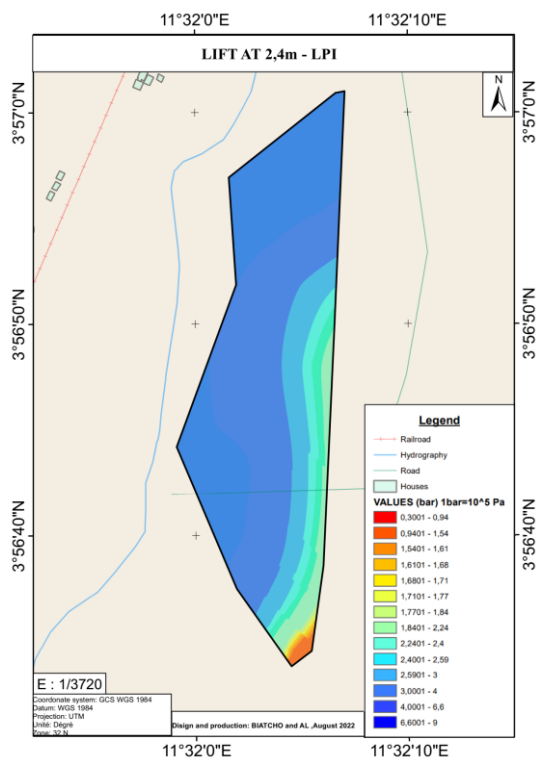


Figure 7 – Ground bearing capacity to a depth of 2.4m (LPI).

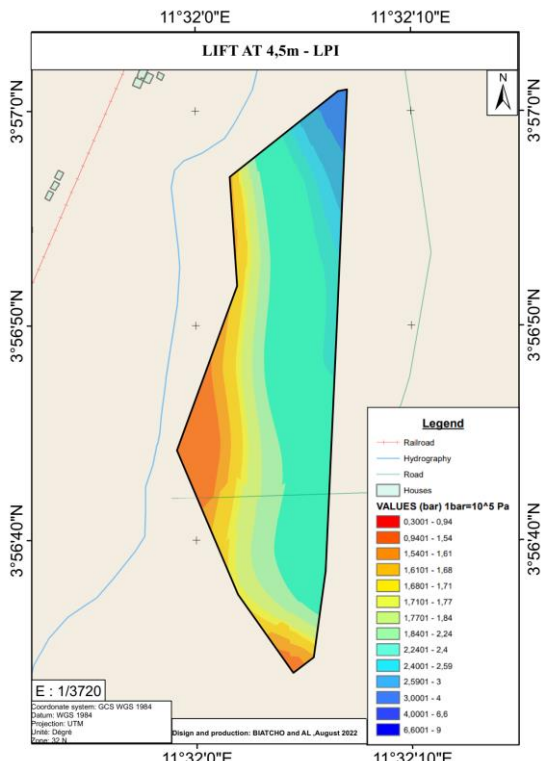


Figure 9 – Soil bearing capacity to depth 4.5m (LPI).

3.3 Mapping of soil bearing capacity following the KBE and KO stochastic methods.

3.3.1 Ground bearing capacity to a depth of 1.2m.

Figures (10) and (11) illustrate the mapping of ground bearing capacity at a depth of 1.2 m according to the KO and KBE methods.

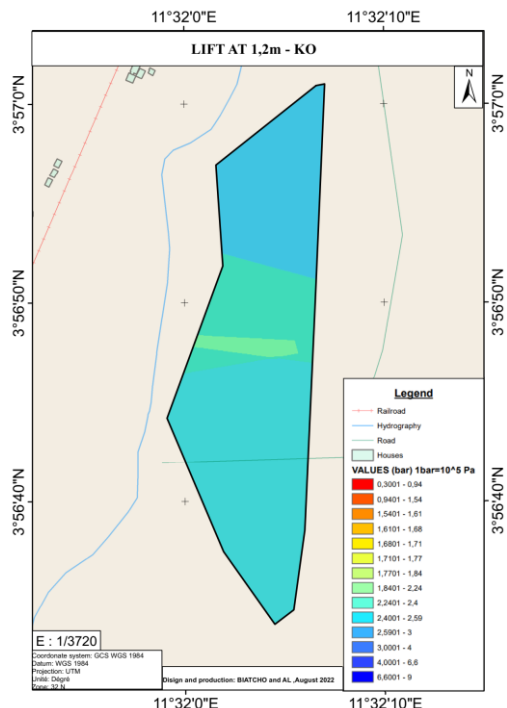


Figure 10– Ground bearing capacity to a depth of 1.2m (KO).

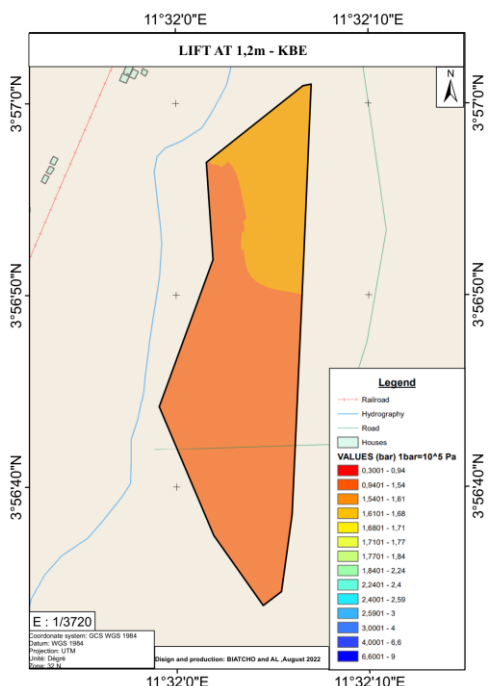


Figure 11 – Ground bearing capacity to a depth 1.2m (KBE).

3.3.2 Ground bearing capacity to a depth of 2.4m.

Figures (12) and (13) illustrate the mapping of ground bearing capacity at a depth of 2.4 m using the KO and KBE methods.

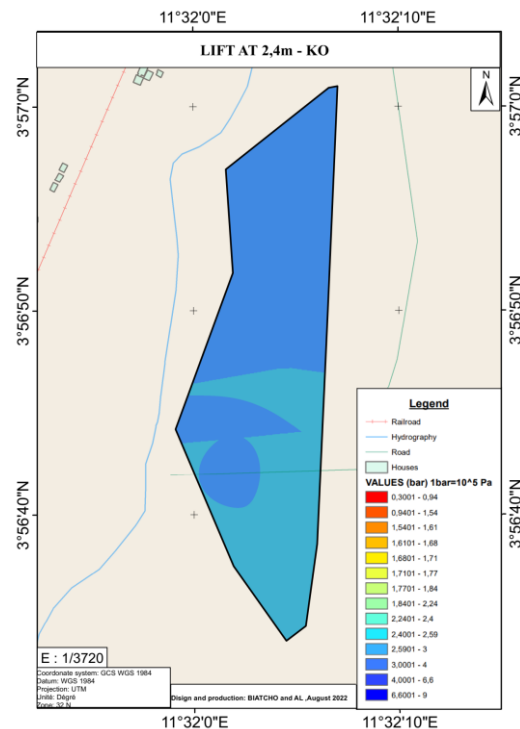


Figure 12 – Ground bearing to a depth of 2.4m (KO).

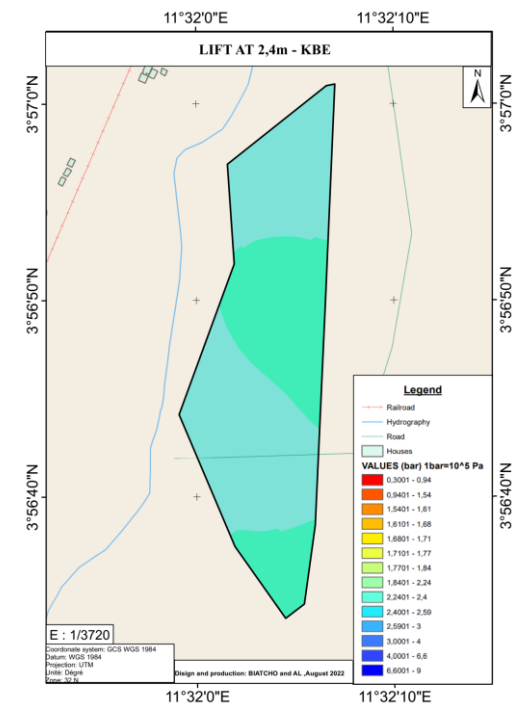


Figure 13 – Ground bearing to a depth of 2.4m (KBE).

3.3.3 Ground bearing capacity to a depth of 4.5m.

Figures (14) and (15) illustrate the mapping of ground bearing capacity at a depth of 4.5 m using the KO and KBE methods.

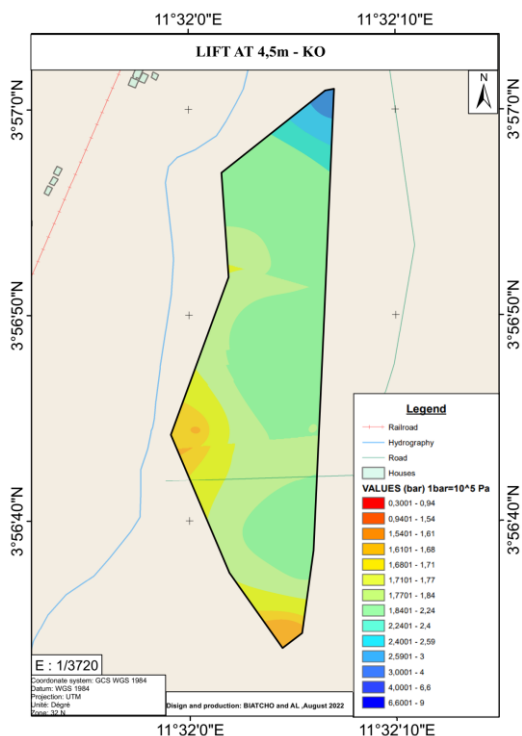


Figure 14 – Ground bearing to a depth of 4.5m (KO)

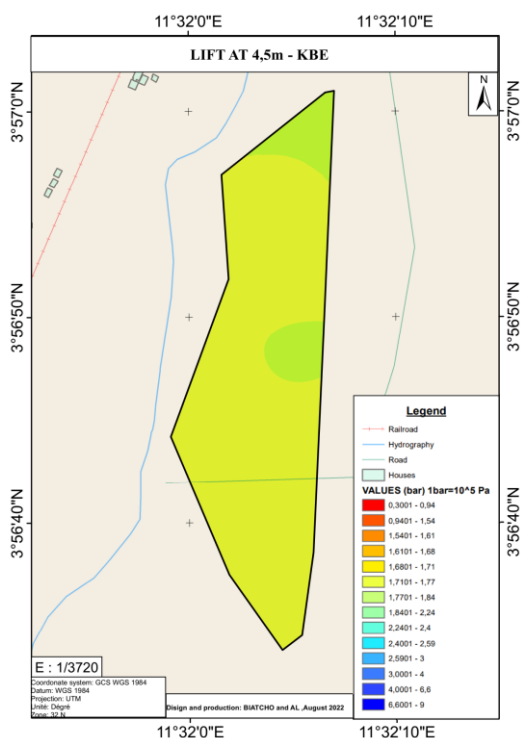


Figure 15 – Ground bearing to a depth of 4.5m (KBE)

3.4 Comparison of the different interpolation methods.

In this part, we have simply reproduced the choices of the study, that is to say to make a comparative analysis between the different deterministic and geostatistical methods to classify the interpolation methods used according to the obtained precisions.

Figure (16) illustrates according to the different methods the values of interpolation error at a depth of 1.2m, 2.4, and 4.5m. We can observe that the interpolation error for stochastic methods remains lower than that of deterministic methods.

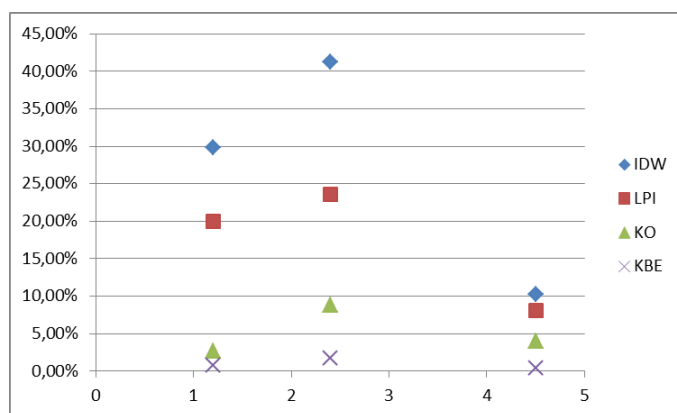


Figure 16 – Interpolation error of bearing capacity at a depth of 1.2m, 2.4, and 4.5m..

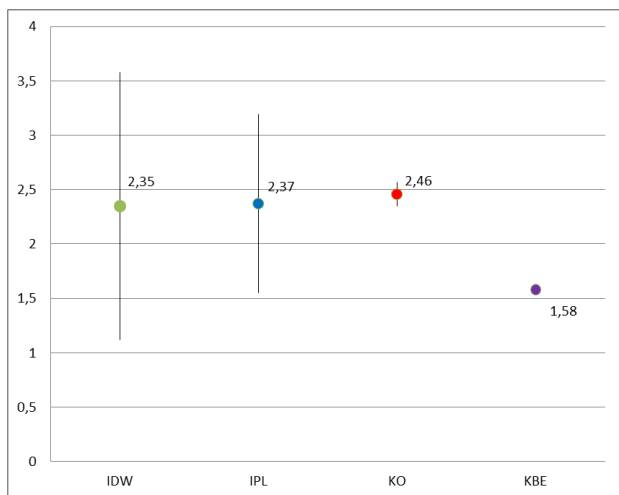
3.4.1 Comparison of the different interpolation methods of ground bearing capacity to a depth of 1.2m.

The average ground bearing capacity at a depth of 1.2 m according to the different interpolation methods is shown in Table (4). It shows that the coefficient of variation of bearing capacity according to the IDW, IPL, KO and KBE methods is 52.34%, 36.60%, 4.47% and 1.90% respectively.

Table 4 – Average ground bearing capacity at a depth of 1.2m.

DETERMINISTIC METHODS				
Interpolation	Type of Variogram	Number of classes	Average	Typical deviation
IDW	/	12	2,35	1,23
LPI	/	12	2,37	0,82
GEOSTATISTICAL METHODS				
K O	Spherical	12	2,46	0,11
KBE	Spherical	12	1,58	0,03

Figure (17) illustrates according to the different methods, the distribution of the ground bearing capacity at a depth of 1.2 m as well as its average value. It shows that the variability of the of the ground bearing capacity of KBE which represents 2.44%, 3.66% and 27.27% of IDW, IPL and KO respectively.



○ Average value of the ground bearing capacity at a depth of 1.2m.

Figure 17 – Distribution of the ground bearing capacity at a depth of 1.2m.

Thus, at a depth of 1.2m we see that the stochastic methods studied are more precise than the deterministic methods and the KBE is the most accurate of all, due to their low interpolation (Figure16) error and low ground bearing variability.

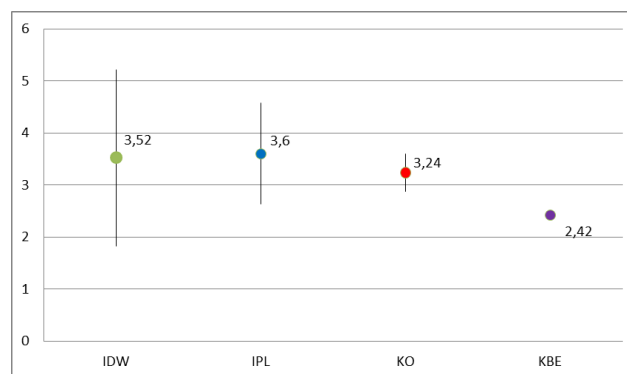
3.4.2 Comparison of the different interpolation methods of ground bearing capacity to a depth of 2.4m.

The average ground bearing capacity at a depth of 2.4 m according to the different interpolation methods is shown in Table (5). It shows that the coefficient of variation of bearing capacity according to the IDW, IPL, KO and KBE methods are 48.30%, 26.94%, 11.11% and 2.89% respectively.

Figure (18) illustrates according to the different methods, the distribution of the ground bearing capacity at a depth of 2.4 m as well as its average value. It shows that the variability of the of the ground bearing capacity of KBE which represents 4.12%, 7.22% and 19.44% of IDW, IPL and KO respectively.

Table 5 – Average ground bearing capacity at a depth of 2.4m.

DETERMINISTIC METHODS				
Interpolation	Type of Variogram	Number of classes	Average	Typical deviation
IDW	/	12	3,52	1,7
LPI	/	12	3,6	0,97
GEOSTATISTICAL METHODS				
K O	Spherical	12	3,24	0,36
KBE	Spherical	12	2,42	0,07



○ Average value of the ground bearing capacity at a depth of 2.4m.

Figure 18 – Distribution of the ground bearing capacity at a depth of 2.4m.

Thus, at a depth of 2.4m we see that the stochastic methods studied are more precise than the deterministic methods and the KBE is the most accurate of all, due to their low interpolation error (Figure16) and low ground bearing variability.

3.4.3 Comparison of the different interpolation methods of ground bearing capacity to a depth of 4.5m.

The average ground bearing capacity at a depth of 4.5 m according to the different interpolation methods is shown in Table (6). It shows that the coefficient of variation of bearing capacity according to the IDW, IPL, KO and KBE methods is 21.69%, 17.28%, 8.47% and 1.14% respectively.

Table 6 – Average ground bearing capacity at a depth of 4.5m.

DETERMINISTIC METHODS				
Interpolation	Type of Variogram	Number of classes	Average	Typical deviation
IDW	/	12	1,89	0,41
LPI	/	12	1,91	0,33
GEOSTATISTICAL METHODS				
K O	Spherical	12	1,89	0,16
KBE	Spherical	12	1,75	0,02

Figure (19) illustrates according to the different methods, the distribution of the ground bearing capacity at a depth of 4.5 m as well as its average value. It shows that the variability of the of the ground bearing capacity of KBE which represents 4.12%, 7.22% and 19.44% of IDW, IPL and KO respectively.

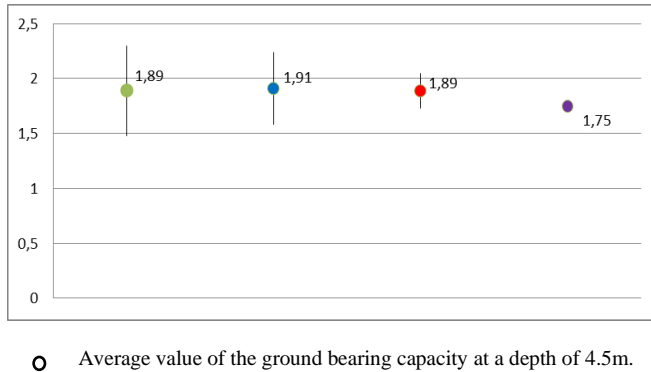


Figure 19 – Distribution of the ground bearing capacity at a depth of 2.4m.

Thus, at a depth of 4.5m we see that the stochastic methods studied are more precise than the deterministic methods and the KBE is the most accurate of all, due to their low interpolation error (Figure16) and low ground bearing variability.

4. Conclusion

This article focused on the comparative study of the reliability of some deterministic and stochastic interpolation methods for the mapping of soil bearing capacity on a site in the locality of Olembé in Cameroon.

Among the stochastic interpolation methods, Ordinary Kriging and Empirical Bayesian Kriging have been examined while Inverse Distance Weighted and Local Polynomial Interpolation have been retained as deterministic methods for estimating ground bearing capacity between different sampling points for respective depths of 1.2m, 2.4m and 4.5m.

Soil bearing capacity maps were developed using ArcGIS software for the different depths using the four interpolation methods. The comparative analysis of the accuracy of the results obtained according to the different interpolation methods made it possible to know that of the interpolation methods studied, the stochastic

methods are more accurate and the empirical Bayesian kriging is the most accurate of all.

References

- Achilleos, G.A., 2011. *The Inverse Distance Weighted interpolation method and error propagation mechanism – creating a DEM from an analogue topographical map*. Journal of Spatial Science, 56(2), pp. 283-304. <http://dx.doi.org/10.1080/14498596.2011.623348>
- AFNOR – Association française de normalisation, 1990. *NF P94-115: Soils investigation and testing. In situ tests. Dynamic penetration B tip*. AFNOR, France.
- AFNOR – Association française de normalisation, 2013. *NF P 94-500: Missions d'ingénierie géotechnique - Classification et spécifications*. AFNOR, France.
- Ambassa, Z. and Amba, J.C., 2020. *Assessment of stiffness and strength parameters for the soft soil Model of clays of Cameroon*. Hindawi, International Journal Advances in Civil Engineering, 2020, pp.1-16. <https://doi.org/10.1155/2020/8877367>
- Baillargeon. S., Pouliot. J, Rivest. J. P, Fortin. V & Fitzback. J., 2004. *Interpolation statistique multivariable de données de précipitations dans un cadre de modélisation hydrologique*. Colloque Géomatique, Montréal.
- Baillargeon. S., 2005. *Le krigeage : revue de la théorie et application à l'interpolation spatiale de données de précipitations*. Faculté des Sciences et de Genie, Université Laval, QUEBEC.
- Bosser. P., 2011. *Interpolation spatiale*. Ecole Nationale des Sciences Géographiques, France.
- Boulier, F., 2020. *Calcul Numérique, Interpolation, Intégration, Équations Différentielles*. Polytech Lille, France.
- ESRI - Environmental Systems Research Institute, 2022. *Empirical Bayesian Kriging*. [online] Available at: <<https://desktop.arcgis.com/fr/arcmap/10.4/extensions/geostatistical-analyst/what-is->

empirical-bayesian-kriging-.htm>. [Accessed 6 September 2022].

Fotios, M., Kyriaki, K., Panagiotis, A., and Simeon, P., 2013. *Kriging Interpolation Method for Estimation of Continuous Spatial Distribution of Precipitation in Cyprus*. British Journal of Applied Science & Technology, 3(4), pp. 1286-1300.

Koussa, M., 2018. *Etude comparative entre les méthodes d'interpolation pour la cartographie des Nitrates: Cas d'application les eaux souterraines de Djelfa, Algérie*. Agriculture and Forestry journal, 2(1), pp. 18-25.

Leborgne, G., 2021. *Interpolation polynomiale, intégration numérique, résolution numérique d'équations différentielles*, ISIMA, France.

Leroux, R., 2007. *Définition et application d'une méthodologie pour l'étude de la propagation d'états de mer en milieu cotier*.

Rebai, N., Slama, T. and Turk, M.M., 2007. *Evaluation de différentes méthodes d'interpolation spatiale pour la production d'un MNT à partir de données topographiques dans un SIG*. Revue XYZ, (110), pp. 19-28.

Setianto, A. and Triandini, T., 2013. *Comparison of kriging and inverse distance weighted (IDW) interpolation methods in lineament extraction and analysis*. Journal of Applied Geology, 5(1), pp.21-29.

<https://doi.org/10.22146/jag.7204>

Zhengquan, L., Kuo, W., Hao, M.Y.W., 2018. *An adjusted inverse distance weighted spatial interpolation method*. In: 3rd International Conference on Communications, Information Management and Network Security.